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Time-dependent harmonic oscillators in an electromagnetic field

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Abstract. We consider the most general form of time-dependent charged harmonic oscillators in an electromagnetic field. The quantum mechanical solution is developed and the Green function is derived. Furthermore, we use gauge-invariant formulation of quantum mechanics to obtain the transition probabilities in the electric dipole approximation.

1. Introduction

Recently, there has been considerable interest in time-dependent harmonic oscillators (TDHO), such as the variable-mass oscillator (which, e.g., arises in the Fabry–Perot cavity) and the variable-frequency oscillator (e.g. which arises in the slowly lengthening pendulum) (Colegrave and Abdalla 1981, 1982, 1983, Leach 1983). Landovitz *et al* (1979) has presented a formalism for TDHO, developed a quantum mechanical solution and derived a Green function. The behaviour of TDHO in an electromagnetic field is also a very important problem in practice. In this paper, we shall generalise Landovitz's formalism to treat this problem. For simplicity we use the electric dipole approximation (EDA) (Kobe 1982). Furthermore, in order to obtain the transition probabilities of TDHO in the EDA, we shall use the gauge-invariant formulation of quantum mechanics (GIF) (Yang 1976, 1982, 1983, 1985, Kobe and Smirl 1978, Kobe and Wen 1982, Kobe 1984).

2. Generalised Landovitz's formalism

The Hamiltonian of TDHO in the Coulomb gauge in the EDA is

$$H = f(t)[\boldsymbol{p} - (q/c)\boldsymbol{A}(t)]^2/2m + g(t)\tfrac{1}{2}m\omega_0^2 x^2 \tag{1}$$

where $\boldsymbol{A}(t)$ is the vector potential at the origin, which is chosen to be a transverse field; the scalar potential A_0 is chosen to be zero. The Hamiltonian equations yield

$$\dot{x} = \frac{\partial H}{\partial p} = f(t)\left(p - \frac{q}{c}A\right)m^{-1} = f(t)\Pi/m \tag{2}$$

$$\dot{\Pi} = \dot{p} - \frac{q}{c}\dot{A} = -\frac{\partial H}{\partial x} - \frac{q}{c}\dot{A} = -g(t)m\omega_0^2 x + qE \tag{3}$$

where Π is the mechanical momentum. Landovitz *et al* introduced the operators $\hat{O}_+$ and $\hat{O}_-$, which are defined by

$$\hat{O}_+ = U^\dagger \hat{O} U \qquad \hat{O}_- = U \hat{O} U^\dagger \tag{4}$$

where U is the evolution operator (Cohen-Tannoudji *et al* 1977). When there is no electromagnetic field, $\boldsymbol{A}(t) = 0$, $\Pi = p$, Landovitz *et al* made the following assumption:

$$\hat{x}_+ = a(t)\hat{x} + b(t)\hat{p} \qquad \hat{p}_+ = c(t)\hat{x} + d(t)\hat{p}. \tag{5}$$

Now, when there is an electromagnetic field we shall generalise (5) to

$$\hat{x}_+ = a(t)\hat{x} + b(t)\hat{\Pi} + u(t) \qquad \hat{\Pi}_+ = c(t)\hat{x} + d(t)\hat{\Pi} + v(t) \tag{6}$$

where

$$a(0) = d(0) = 1 \qquad b(0) = c(0) = u(0) = v(0) = 0. \tag{7}$$

When there is no electromagnetic field

$$u(t) = v(t) = 0 \tag{8}$$

and (6) reduces to (5).

For H expressed by (1), x and p are taken as operators satisfying the commutation relation

$$[\hat{x}, \hat{p}] = \mathrm{i}\hbar. \tag{9}$$

From

$$\mathrm{i}\hbar \frac{\partial U}{\partial t} = HU \tag{10}$$

we obtain

$$\frac{\mathrm{d}\hat{O}_+}{\mathrm{d}t} = \frac{1}{\mathrm{i}\hbar}[\hat{O}_+, \hat{H}_+] + \left(\frac{\partial \hat{O}}{\partial t}\right)_+. \tag{11}$$

Specifically

$$\frac{\mathrm{d}\hat{x}_+}{\mathrm{d}t} = \frac{1}{\mathrm{i}\hbar}[\hat{x}_+, \hat{H}_+] \tag{12}$$

$$\frac{\mathrm{d}\hat{p}_+}{\mathrm{d}t} = \frac{1}{\mathrm{i}\hbar}[\hat{p}_+, \hat{H}_+] \tag{13}$$

$$\frac{\mathrm{d}^2\hat{x}_+}{\mathrm{d}t^2} = \frac{1}{\mathrm{i}\hbar}\left[\frac{\mathrm{d}\hat{x}_+}{\mathrm{d}t}, \hat{H}_+\right] + \frac{\partial}{\partial t}\left(\frac{\mathrm{d}\hat{x}_+}{\mathrm{d}t}\right). \tag{14}$$

Also (4) and (9) imply

$$[\hat{x}_+, \hat{p}_+] = \mathrm{i}\hbar. \tag{15}$$

Application of (12)-(15) to the Hamiltonian H in (1) yields respective equations identical to (2)-(3) with x replaced by the operator $\hat{x}_+$ and p replaced by the operator $\hat{p}_+$ and, Π replaced by the operator $\hat{\Pi}_+$. By assumption (6) and the corresponding operator equations (2)-(3) we obtain

$$c(t) = \frac{m}{f}\frac{\mathrm{d}a}{\mathrm{d}t} \qquad d(t) = \frac{m}{f}\frac{\mathrm{d}b}{\mathrm{d}t} \qquad v(t) = \frac{m}{f}\frac{\mathrm{d}u}{\mathrm{d}t}. \tag{16}$$

The condition

$$[\hat{x}_+, \hat{\Pi}_+] = [\hat{x}, \hat{\Pi}] \tag{17}$$

requires

$$ab - bc = 1. \tag{18}$$

Assuming the oscillator to be in a state $|n\rangle$ at $t = 0$, we compute the expectation values of $\hat{x}^2$ and $\dot{\hat{x}}^2$ at a later time t, denoted by $\langle \hat{x}_+^2 \rangle_n$ and $\langle \dot{\hat{x}}_+^2 \rangle_n$, respectively. By means of annihilation and creation operators we obtain

$$\langle n|\hat{x}^2|n\rangle = (n+\tfrac{1}{2})\hbar/m\omega_0 \qquad \langle n|\hat{p}^2|n\rangle = (n+\tfrac{1}{2})m\hbar\omega_0 \qquad \langle n|\hat{x}\hat{p}+\hat{p}\hat{x}|n\rangle = 0. \tag{19}$$

The operator $\dot{\hat{x}}_+$ is related to $\hat{\Pi}_+$ through (2), i.e.

$$\dot{\hat{x}}_+ = f(t)\hat{\Pi}_+/m. \tag{20}$$

Using (6), (19) and (20), we obtain

$$\langle \hat{x}^2 \rangle_n = (a^2 + m^2\omega_0^2 b^2)(n+\tfrac{1}{2})\hbar/m\omega_0 + (u - bqA/c)^2 \tag{21}$$

$$\langle \dot{\hat{x}}_+^2 \rangle_n = f^2[(1/m^2\omega_0^2)c^2 + d^2](n+\tfrac{1}{2})\hbar\omega_0/m + (f^2/m^2)(v - dqA/c)^2. \tag{22}$$

Now we shall find the six functions a, b, c, d, u, v in (6) for a particular case, the dissipative harmonic oscillator. In this case

$$f(t) = \exp(-t/\tau) \qquad g(t) = \exp(t/\tau) \tag{23}$$

the equation of motion is

$$\ddot{x} + (1/\tau)\dot{x} + \omega_0^2 x - (q/m)\exp(-t/\tau)E = 0. \tag{24}$$

From (24) we obtain the operator equation of $\hat{x}_+$ by replacing x with $\hat{x}_+$. For the electric field

$$E = E_0 \sin(\Omega t - \phi) \tag{25}$$

the solution obtained is

$$\hat{x}_+ = \exp(-t/2\tau)(O_1 \cos\omega t + O_2 \sin\omega t) + B\exp(-t/\tau)\sin\Omega t \tag{26}$$

where

$$\omega = (\omega_0^2 - 1/4\tau^2)^{1/2} \qquad B = \frac{qE_0}{m[(\omega_0^2-\Omega^2)^2 + \Omega^2/\tau^2]^{1/2}} \qquad \phi = \tan^{-1}\frac{\Omega/\tau}{\omega_0^2 - \Omega^2} \tag{27}$$

and O_1 and O_2 are operators to be determined. From (20)

$$\hat{\Pi}_+ = \exp(t/\tau) m \frac{\mathrm{d}\hat{x}_+}{\mathrm{d}t}. \tag{28}$$

At $t = 0$

$$\hat{x}_+ = \hat{x} \qquad \frac{\mathrm{d}\hat{x}_+}{\mathrm{d}t} = \frac{\Pi}{m}. \tag{29}$$

Thus

$$O_1 = \hat{x} \qquad O_2 = \frac{1}{\omega}[\hat{\Pi}/m + (1/2\tau)\hat{x}] - B\frac{\Omega}{\omega} \tag{30}$$

and hence we obtain

$$a = \exp(-t/2\tau)[\cos \omega t + (1/2\omega\tau) \sin \omega t] \qquad b = \exp(-t/2\tau)\left(\frac{1}{m\omega}\right) \sin \omega t$$

$$c = -\exp(t/2\tau)m[\omega + (1/4\omega\tau^2)] \sin \omega t \qquad d = \exp(t/2\tau)[\cos \omega t - (1/2\omega\tau) \sin \omega t] \tag{31}$$

$$u = (B/\omega) \exp(-t/\tau)[\omega \sin \Omega t - \Omega \exp(t/2\tau) \sin \omega t]$$

$$v = (B\Omega/\tau) \exp(-t/\tau)[\tau \cos \Omega t - (1/\Omega) \sin \Omega t + (1/2\omega) \exp(t/2\tau) \sin \omega t - \tau \exp(t/2\tau) \cos \omega t].$$

From (31) we know that four functions a, b, c, d are the same as that obtained by Landovitz *et al* in the case of absence of an electromagnetic field and two functions u and v disappear as expected when there is no electromagnetic field. Similarly, we can obtain the results corresponding to the three other particular cases discussed by Landovitz *et al.* Of course, when there is no electromagnetic field, all our results reduce to that of Landovitz.

3. Green function

The Green function is defined by (Cohen-Tannoudji *et al* 1977)

$$\langle x|U|x'\rangle = G(x, x'; t). \tag{32}$$

The wavefunction $\psi(x, t)$ is obtainable from $\psi(x, 0)$ by the formula

$$\psi(x, t) = \int_{-\infty}^{\infty} G(x, x'; t)\psi(x', 0)\, dx'. \tag{33}$$

The boundary condition

$$\lim_{t\to 0} \psi(x, t) = \psi(x, 0) \tag{34}$$

implies

$$\lim_{t\to 0} G(x, x'; t) = \delta(x' - x). \tag{35}$$

From (32)

$$\langle x|U\hat{x}_+|x'\rangle = \langle x|\hat{x}U|x'\rangle = xG(x, x'; t) \tag{36}$$

$$\langle x|U\hat{p}|x'\rangle = i\hbar(\partial/\partial x')G(x, x'; t) \tag{37}$$

$$\langle x|U\hat{\Pi}|x'\rangle = i\hbar(\partial/\partial x')G(x, x'; t) - (qA/c)G(x, x'; t). \tag{38}$$

Substituting (6) in (36) and using (38) we obtain

$$\frac{\partial G}{\partial x'} = \left(\frac{i}{\hbar b}\right)\left(ax' - x - \frac{bqA}{c} + u\right)G. \tag{39}$$

Thus we obtain

$$G = g(x, t) \exp\left[\left(\frac{i}{\hbar b}\right)\left(\tfrac{1}{2}ax'^2 - xx' - \frac{bqA}{c}x' + ux'\right)\right]. \tag{40}$$

By the following formulae

$$\langle x|\hat{x}_- U|x'\rangle = \langle x|U\hat{x}|x'\rangle = x'G(x, x'; t) \tag{41}$$

$$\langle x|\hat{\Pi} U|x'\rangle = \frac{\hbar}{\mathrm{i}}\frac{\partial}{\partial x}G(x, x'; t) - \frac{qA}{c}G(x, x'; t) \tag{42}$$

$$\hat{x}_- = \mathrm{d}\hat{x} - b\hat{\Pi} - (ud - vb) \tag{43}$$

we obtain

$$\frac{\partial G}{\partial x} = \frac{\mathrm{i}}{\hbar b}\left(\mathrm{d}x - x' + \frac{bqA}{c} - ud + vb\right)G. \tag{44}$$

From (40)

$$\frac{\partial g(x, t)}{\partial x} = \frac{\mathrm{i}d}{\hbar b}\left(x + \frac{bqA}{cd} - u + \frac{vb}{d}\right). \tag{45}$$

Thus

$$g(x, t) = F(t)\exp\left[\left(\frac{\mathrm{i}d}{2\hbar b}\right)\left(x^2 + \frac{2bqA}{cd}x - 2ux + \frac{2vb}{d}x\right)\right]. \tag{46}$$

Hence

$$G(x, x'; t) = F(t)\exp\left[\left(\frac{\mathrm{i}}{2\hbar b}\right)\right.$$
$$\left.\times\left(\mathrm{d}x^2 + ax'^2 - 2xx' + \frac{2bqA}{c}(x - x') - 2(ud - vb)x + 2ux'\right)\right]. \tag{47}$$

The expression for $F(t)$ can be obtained from (34); the result is

$$F(t) = 1/(2\pi\mathrm{i}\hbar b)^{1/2}. \tag{48}$$

Therefore

$$G(x, x'; t) = \left(\frac{\beta}{\mathrm{i}\pi}\right)^{1/2}\exp\left[\mathrm{i}\beta\left(\mathrm{d}x^2 + ax'^2 - 2xx' + \frac{2bqA}{c}(x - x') - 2(ud - vb)x + 2ux'\right)\right] \tag{49}$$

where

$$\beta = \frac{1}{2\hbar b}. \tag{50}$$

For an operator $\hat{O}$, the expectation value at time t is

$$\langle O\rangle_t = \int_{-\infty}^{\infty}\psi^+(x, t)\hat{O}\psi(x, t)\,\mathrm{d}x$$
$$= \int_{-\infty}^{\infty}\mathrm{d}x\int_{-\infty}^{\infty}\mathrm{d}x'\int_{-\infty}^{\infty}\mathrm{d}x''\,\psi^+(x'', 0)G^+(x, x''; t)\hat{O}G(x, x'; t)\psi(x', 0). \tag{51}$$

When there is no electromagnetic field, $A = u = v = 0$, (49) reduces to the result given by Landovitz *et al.* For the case of a strongly pulsating mass, it reduces to the result given by Colegrave and Abdalla (1983).

Now we are going to calculate expectation values of some important quantities by equation (51). If the oscillator is in a state $n=0$ at $t=0$, then we can obtain the expectation values of operators $\hat{x}$ and $\hat{\Pi}$ through quite a tedious calculation:

$$\langle\hat{x}\rangle = u - bqA/c \tag{52}$$

$$\langle\hat{\Pi}\rangle = v - dqA/c. \tag{53}$$

When there is no electromagnetic field, the above two expectation values are equal to zero as expected.

4. Transition probability

Recently, Yang and Kobe have developed GIF for obtaining gauge-independent probability amplitudes. Now we shall use GIF to discuss TDHO in EDA. The dark Hamiltonian of TDHO is (Landovitz 1979)

$$H_0 = f(t)p^2/2m + g(t)\tfrac{1}{2}m\omega_0^2x^2. \tag{54}$$

In general, in an electromagnetic field the Hamiltonian is

$$H = f(t)(\boldsymbol{p} - q\boldsymbol{A}/c)^2/2m + g(t)\tfrac{1}{2}m\omega_0^2x^2 + qA_0. \tag{55}$$

Yang's energy operator is (Yang 1976)

$$\mathscr{E} \equiv H - qA_0 = f(t)(\boldsymbol{p} - q\boldsymbol{A}/c)^2/2m + g(t)\tfrac{1}{2}m\omega_0^2x^2. \tag{56}$$

In the Coulomb gauge in the EDA the Hamiltonian is given by (1) and in this case the energy operator is the same as the Hamiltonian. The eigenvalue problem for the energy operator is

$$[f(t)(\boldsymbol{p} - qA(t)/c)^2/2m + g(t)\tfrac{1}{2}m\omega_0^2x^2]\psi_n = \varepsilon_n\psi_n. \tag{57}$$

When we make the gauge transformation with the gauge function (Kobe and Wen 1982)

$$\Lambda(x, t) = -A(t)x \tag{58}$$

new potentials are

$$A' = 0 \qquad A_0' = -E(t)x \tag{59}$$

the gauge-transformed energy eigenvalue problem is

$$\left(\frac{1}{2m}f(t)p_x^2 + g(t)\tfrac{1}{2}m\omega_0^2x^2\right)\psi_n' = \varepsilon_n\psi_n'. \tag{60}$$

When we make the following transformations:

$$M = \frac{m}{f(t)} \qquad \omega^2 = \omega_0^2 f(t)g(t) \tag{61}$$

equation (60) becomes

$$\left(\frac{1}{2M}p_x^2 + \frac{1}{2}M\omega^2x^2\right)\psi_n' = \varepsilon_n\psi_n'. \tag{62}$$

Comparing (62) with the equation for a free harmonic oscillator, we readily obtain (Merzbacher 1970)

$$\varepsilon_n(t)=(n+\tfrac{1}{2})\hbar\omega=(n+\tfrac{1}{2})(f(t)g(t))^{1/2}\hbar\omega_0 \tag{63}$$

$$\psi'_n=N_n(-1)^n\exp(\xi^2/2)\frac{\mathrm{d}^n}{\mathrm{d}\xi^n}[\exp(-\xi^2)] \tag{64}$$

where

$$\xi=\alpha x=\left(\frac{g^{1/2}m\omega_0}{f^{1/2}\hbar}\right)^{1/2}x \tag{65}$$

$$N_n=\left(\frac{\alpha}{\pi^{1/2}2^n n!}\right)^{1/2}=\left(\frac{g}{f}\right)^{1/8}\left(\frac{m\omega_0}{\pi\hbar}\right)^{1/4}(2^n n!)^{-1/2}. \tag{66}$$

The equation of motion for $c_n(t)=\langle\psi_n|\psi\rangle$ is (Kobe and Smirl 1978)

$$\mathrm{i}\hbar\dot{c}_n-\varepsilon_n c_n=\sum_m\left\langle\psi_n\left|\left(qA_0-\mathrm{i}\hbar\frac{\partial}{\partial t}\right)\psi_m\right.\right\rangle c_m. \tag{67}$$

Since the matrix element in (67) is gauge invariant

$$\langle\psi_n|(qA_0-\mathrm{i}\hbar\,\partial/\partial t)\psi_m\rangle=\langle\psi'_n|(qA'_0-\mathrm{i}\hbar\,\partial/\partial t)\psi'_m\rangle=-qE(t)\langle\psi'_n|x\psi'_m\rangle-\mathrm{i}\hbar\langle\psi'_n|\,(\partial/\partial t)\psi'_m\rangle. \tag{68}$$

The matrix elements of x are

$$\langle\psi'_n|x\psi'_m\rangle=\left(\frac{f}{g}\right)^{1/4}\left(\frac{\hbar}{2m\omega_0}\right)^{1/2}[n^{1/2}\delta_{m,n-1}+(n+1)^{1/2}\delta_{m,n+1}]. \tag{69}$$

Noting N_n and ξ are all functions of time, we obtain

$$\left\langle\psi'_n\left|\frac{\partial}{\partial t}\psi'_m\right.\right\rangle=-u[n(n-1)]^{1/2}\delta_{m,n-2}+u[(n+1)(n+2)]^{1/2}\delta_{m,n+2} \tag{70}$$

where

$$u=\frac{1}{8}\left(\frac{f}{g}\right)\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{g}{f}\right). \tag{71}$$

Equation (67) then becomes

$$\begin{aligned}\mathrm{i}\hbar\dot{c}_n-\varepsilon_n c_n={}&-qE\left(\frac{f}{g}\right)^{1/4}\left(\frac{\hbar}{2m\omega_0}\right)^{1/2}[n^{1/2}c_{n-1}+(n+1)^{1/2}c_{n+1}]\\&+\mathrm{i}\hbar u\{[n(n-1)]^{1/2}c_{n-2}-[(n+1)(n+2)]^{1/2}c_{n+2}\}.\end{aligned} \tag{72}$$

Equation (72) shows that there are transitions from the state n up to the state $n+1$, $n+2$, and down to the state $n-1$, $n-2$. Equation (72) is the general equation for obtaining transition probabilities of TDHO in the EDA. In particular, it can be used to solve variable mass harmonic oscillator problems and variable frequency harmonic oscillator problems. In this paper we only give the approximate solution for dissipative harmonic oscillator in a special case.

For a dissipative harmonic oscillator, using (23) and assuming

$$E(t)=E_0\sin\Omega t \tag{73}$$

we obtain from (72)

$$\mathrm{i}\hbar\dot{c}_n - \varepsilon_n c_n = -\gamma\hbar\omega_0 j(t)[n^{1/2}c_{n-1} + (n+1)^{1/2}c_{n+1}] + \mathrm{i}\hbar k\{[n(n-1)]^{1/2}c_{n-2} - [(n+1)(n+2)]^{1/2}c_{n+2}\} \tag{74}$$

where

$$\varepsilon_n = (n+\tfrac{1}{2})(fg)^{1/2}\hbar\omega_0 = (n+\tfrac{1}{2})\hbar\omega_0 \tag{75}$$

$$\gamma = qE_0(2m\hbar\omega_0^3)^{-1/2} \tag{76}$$

$$j(t) = \left(\frac{f}{g}\right)^{1/4}\frac{E(t)}{E_0} = \exp(-t/2\tau)\sin\Omega t \tag{77}$$

$$k = \frac{1}{8}\left(\frac{f}{g}\right)\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{g}{f}\right) = \frac{1}{4\tau}. \tag{78}$$

In the case $n \leqslant 1$, (74) reduces to

$$\mathrm{i}\hbar\dot{c}_n - \varepsilon_n c_n = -\gamma\hbar\omega_0 j(t)(n^{1/2}c_{n-1} + (n+1)^{1/2}c_{n+1}) - \mathrm{i}\hbar k[(n+1)(n+2)]^{1/2}c_{n+2}. \tag{79}$$

Now we use an approximation such that, for $n \leqslant 3$, the solution of c_n can be written as (Kobe and Wen 1982)

$$c_n(t) = \exp(\mathrm{i}\beta(t))(n!)^{-1/2}[Q(\omega_0 t)]^n \exp(-\tfrac{1}{2}|Q(\omega_0 t)|^2) \tag{80}$$

where the function Q and β are to be determined. When (80) is substituted into (79) we obtain

$$(\mathrm{i}\dot{Q} + \gamma\omega_0 j(t) - \omega_0 Q)Q^{-1}nc_n = \left(\tfrac{1}{2}\omega_0 + \dot{\beta} + \mathrm{i}\frac{1}{2}\frac{\mathrm{d}|Q|^2}{\mathrm{d}t} - \gamma\omega_0 j(t)Q - \mathrm{i}kQ^2\right)c_n. \tag{81}$$

For this equation to be valid for $n = 0, 1$, it is necessary that both sides vanish identically:

$$\mathrm{i}\dot{Q} + \gamma\omega_0 j(t) - \omega_0 Q = 0 \tag{82}$$

$$\tfrac{1}{2}\omega_0 + \dot{\beta} + \mathrm{i}\frac{1}{2}\frac{\mathrm{d}|Q|^2}{\mathrm{d}t} - \gamma\omega_0 j(t)Q - \mathrm{i}kQ^2 = 0. \tag{83}$$

Noting that $\mathrm{Im}\, j(t) = 0$, from (83) we obtain

$$\mathrm{Im}\,\beta = \int_0^t \mathrm{Re}(kQ^2)\,\mathrm{d}t. \tag{84}$$

The values of probability $|c_n(t)|^2$ as a function of $\omega_0 t$ when $\omega_0 = \Omega$ and $\omega_0\tau = 2$ for $n = 0, 1, 2, 3$ are given in table 1. c_4 can be obtained from c_0, c_1, c_2, c_3 by letting $n = 2$

Table 1.

Probability	$\omega_0 t$							
	0	0.50	0.75	1.00	1.25	1.50	1.75	2.00
$\|c_0\|^2 = \exp(-2\,\mathrm{Im}\,\beta - \|Q\|^2)$	1	0.979 89	0.945 70	0.877 15	0.775 94	0.638 30	0.554 66	0.430 63
$\|c_1\|^2 = \|Q\|^2\|c_0\|^2$	0	0.012 28	0.051 56	0.102 63	0.222 42	0.300 64	0.363 20	0.389 66
$\|c_2\|^2 = \frac{1}{2}\|Q\|^4\|c_0\|^2$	0	0.000 08	0.001 41	0.014 80	0.031 88	0.070 80	0.118 92	0.176 29
$\|c_3\|^2 = \frac{1}{6}\|Q\|^6\|c_0\|^2$	0	0	0.000 03	0.000 71	0.003 05	0.011 11	0.025 96	0.053 17

in (72). Then we can obtain c_5 by letting $n = 3$ in (72), and so on. The above method of calculating probability is approximate. We have used (82) and (83) to determine Q and β, but (82) and (83) are obtained for $n = 0$, 1 and we use these results to express c_2 and c_3.

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